Circuits with Op Amps

The **operational amplifier (op amp)** is a complex nonlinear device with three distinct operating regions: a linear region, in which the output voltage is proportional to the difference between the two input voltages, and two saturation regions where the output voltage takes on either the positive power supply voltage or the negative power supply voltage. But with two simplifying assumptions about the behavior and characteristics of op amps we can readily analyze circuits containing these devices. The assumptions are as follows:

- The op amp is ideal. From the standpoint of the op amp as a device, this means that the op amp has infinite input resistance, infinite open loop gain, and zero output resistance. From the standpoint of analyzing a circuit containing an op amp, this means that there is no current flowing into the input terminals of the op amp (because of the infinite input resistance) and there is no voltage drop across the input terminals of the op amp when it is operating in its linear region (because of the infinite open loop gain).

- The op amp is operating in its linear region. In order to make this assumption, the circuit containing the op amp must have a **negative feedback path** which is a connection from the output terminal of the op amp to the inverting input terminal (the one labeled with a $-$ sign). This assumption, combined with the assumption that the op amp is ideal, allows us to assume that there is no difference between the voltages at the input terminals of the op amp. This assumption leads to the conclusion that the output voltage must be within the range of voltage values established by the positive and negative power supplies if the op amp is indeed within its linear operating region.

To analyze a circuit with an op amp, we begin be making the above assumptions. Then we write one or more node voltage equations at the input terminals of the op amp. Note that we can never write a node voltage equation at the output terminal of the op amp, because we have no method for calculating the current flowing into the op amp at the output terminal. Once we solve the node voltage equations, we check to see whether or not our second assumption can be validated. To do this, we check the voltage at the output of the op amp to see whether or not its value is within the range of values established by the positive and negative power supplies. If the output voltage is within the specified range, our analysis is complete; if not, the output voltage saturates at the power supply voltage closest to the one calculated in our analysis.

We divide the analysis of a circuit containing an op amp into four steps:

1. Assume that the op amp is ideal and operating in its linear region. Label the two input nodes for the op amp with voltages, usually v_p for the non-inverting terminal (the one with the + sign) and v_n for the inverting terminal (the one with the − sign). Label the output node for the op amp with a voltage, usually v_o.

2. If possible, calculate the numerical value of the node voltage at the non-inverting input to the op amp. Remember that the ideal op amp assumption tells us that there is no current flowing into the op amp. If a numerical calculation is not possible, calculate the node voltage at the non-inverting terminal as a function of the source voltage or voltages connected to that terminal

3. Now, write a KCL equation at the inverting input terminal of the op amp. Remember that by assumption, the voltage at the inverting input node is the same as the voltage at the non-inverting input node calculated in Step 2. The node voltage equation written at the inverting terminal will always involve the output voltage variable because of the negative feedback path that allows the op amp to operate within its linear region. Then solve the node voltage equation for the voltage at the output node.

4. Examine the value of the voltage at the output node. If the op amp is actually operating within its linear region, the output voltage will be between the two power supply voltages. If it is, your analysis is complete. If it is not, then the output voltage is not the value you calculated, but instead will saturate at the power supply voltage it is closest to, giving the correct value for the output voltage.

We illustrate this method with the two examples that follow.

Op Amp Example 1

Find the voltage drop v_o for the circuit in Fig. 1.

Solution

1. We assume that the op amp is ideal and operating in its linear region. This allows us to assume that the value of the current flowing into the

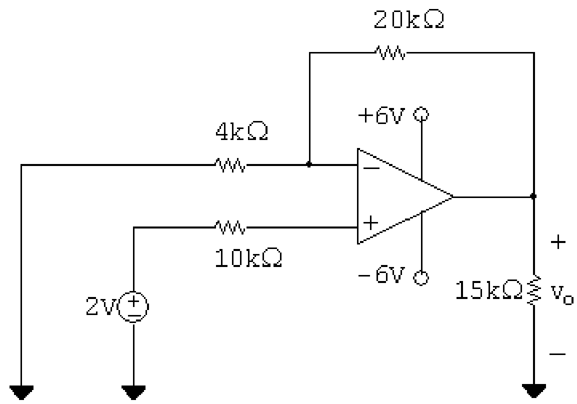

Figure 1: The circuit for Op Amp Example 1

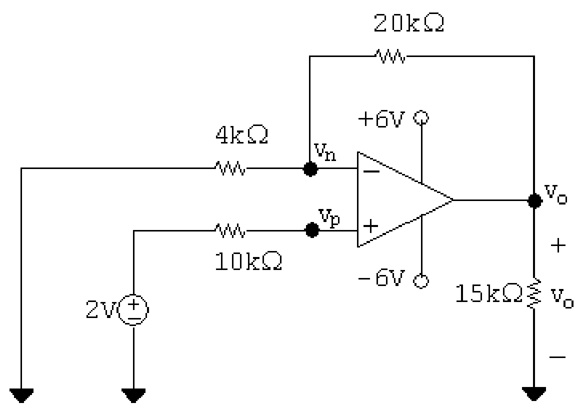

Figure 2: The circuit for Op Amp Example 1, with the three node voltages for the op amp identified and labeled.

op amp at its two input terminals is zero, and that the voltage drop between those same two input terminals is zero. We label the three node voltages, two at the op amp's input and one at its output, as shown in Fig. 2.

2. Calculate the value of the voltage at the non-inverting input node of the op amp. We can do this for the voltage v_p quite easily. Since there is no current flowing into the op amp by assumption, there can be no voltage drop across the 10kΩ resistor. Thus,

$$v_p = 2 \text{ V}.$$

3. Write a node voltage equation at the inverting input node of the op amp. The node voltage equation is written by summing the currents leaving the node v_n. Remember that the current leaving this node and flowing into the op amp is zero by assumption.

$$\frac{v_n - 0}{4000} + \frac{v_n - v_o}{20{,}000} = 0$$

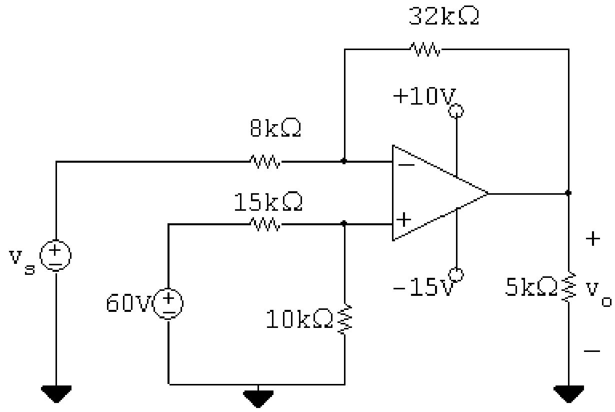

Figure 3: The circuit for Op Amp Example 2

By assumption, there is no voltage drop between the two input terminals for the op amp. Thus,

$$v_n = v_p = 2 \text{ V}.$$

Substituting this value into the node voltage equation and solving for v_o we get

$$v_o = 12 \text{ V}.$$

4. Examine the value of the output voltage. If the op amp is within its linear region of operation, as we assumed, then

$$-6 \text{ V} \leq v_o \leq +6 \text{ V}.$$

But our calculation gave the result $v_o = 12$ V. Therefore, the assumption of linear operation is invalid, and in fact, the op amp has saturated. The value of the output voltage is the same as the value of the power supply closest to the value of 12 V. Thus,

$$v_o = 6 \text{ V}.$$

Op Amp Example 2

Find the range of values for the voltage v_s such that the output voltage v_o does not saturate for the circuit in Fig. 3.

Solution

1. We assume that the op amp is ideal and operating in its linear region. This allows us to assume that the value of the current flowing into the op amp at its two input terminals is zero, and that the voltage drop between those same two input terminals is zero. We label the three node voltages, two at the op amp's input and one at its output, as shown in Fig. 4.

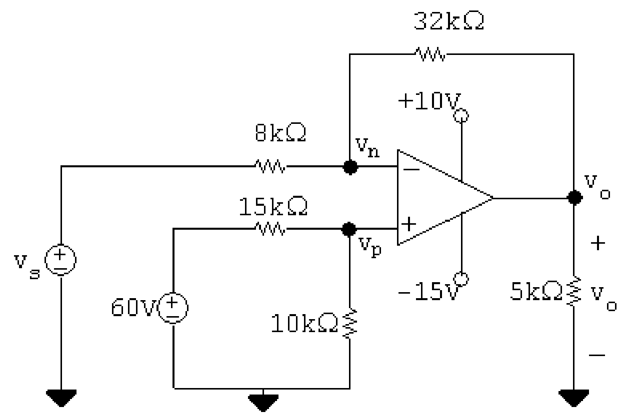

Figure 4: The circuit for Op Amp Example 2, with the three node voltages for the op amp identified and labeled.

2. Calculate the value of the voltage at the non-inverting input node of the op amp. We can do this for the voltage v_p using voltage division. Since there is no current flowing into the op amp by assumption, the wire connecting the node labeled v_p to the op amp acts like an open circuit. Therefore, the loop formed by the 60V source, the 15kΩ resistor and the 10kΩ resistor acts as though it is not attached to the rest of the circuit. The voltage v_p is then the voltage drop across the 10kΩ resistor, whose value we calculate using voltage division. Thus,

$$v_p = \frac{10{,}000}{10{,}000 + 15{,}000}(60) = 24 \text{ V}.$$

3. Write a node voltage equation at the inverting input node of the op amp. The node voltage equation is written by summing the currents leaving the node v_n. Remember that the current leaving this node and flowing into the op amp is zero by assumption.

$$\frac{v_n - v_s}{8000} + \frac{v_n - v_o}{32{,}000} = 0$$

By assumption, there is no voltage drop between the two input terminals for the op amp. Thus,

$$v_n = v_p = 24 \text{ V}.$$

Substituting this value into the node voltage equation and solving for v_o we get

$$v_o = 120 - 4v_s$$

4. Now we use the two power supply voltages as the limiting values for v_0. We consider one limiting value at a time by substituting it into

the equation from Step 3 and calculating the value of v_s that would produce this limiting value. When $v_o = 10\text{V}$,

$$10 = 120 - 4v_s \qquad \text{so} \qquad v_s = \frac{120 - 10}{4} = 27.5 \text{ V}.$$

When $v_o = -15\text{V}$,

$$-15 = 120 - 4v_s \qquad \text{so} \qquad v_s = \frac{120 + 15}{4} = 33.75 \text{ V}.$$

Thus, the range of values for v_s for which v_o will not saturate (and the op amp remains in its linear operating region) is

$$27.5 \text{ V} \leq v_s \leq 33.75 \text{ V}.$$

Now you are ready to practice analyzing circuits with op amps in the problems that follow.

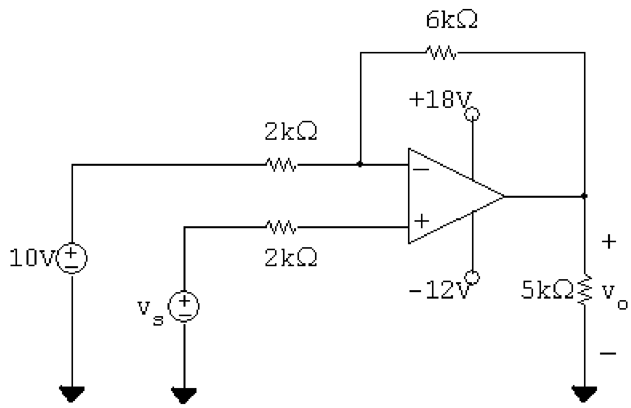

Figure 5: The circuit for Op Amp Practice Problem 1.

Op Amp Practice Problem 1

Find the range of values for the voltage v_s such that the output voltage v_o does not saturate and the op amp remains in its linear region of operation for the circuit in Fig. 5.

1. Assume that the op amp is ideal and operating in its linear region. Label the three node voltages, two at the op amp's input and one at its output, in Fig. 5.

2. Calculate the value of the voltage at the non-inverting input node of the op amp, or write an equation for the voltage v_p in terms of the source voltage.

3. Write a node voltage equation at the inverting input node of the op amp. Solve this equation for v_o.

4. Use the two power supply voltages as the limiting values for v_o and calculate the range of values for v_s that will keep v_o within its limiting values.

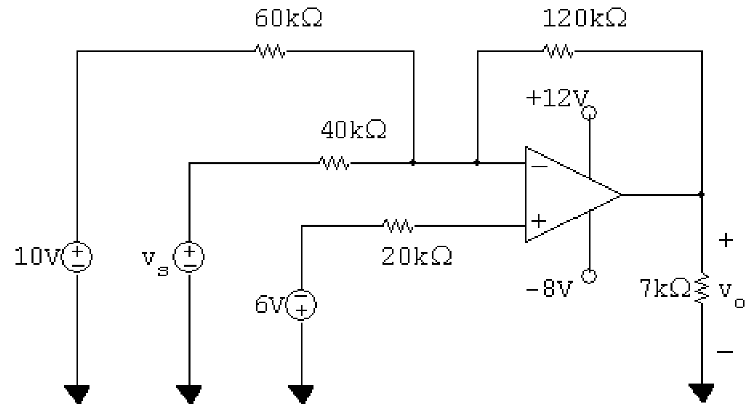

Figure 6: The circuit for Op Amp Practice Problem 2.

Op Amp Practice Problem 2

Find the range of values for the voltage v_s such that the output voltage v_o does not saturate and the op amp remains in its linear region of operation for the circuit in Fig. 6.

1. Assume that the op amp is ideal and operating in its linear region. Label the three node voltages, two at the op amp's input and one at its output, in Fig. 6.

2. Calculate the value of the voltage at the non-inverting input node of the op amp, or write an equation for the voltage v_p in terms of the source voltage.

3. Write a node voltage equation at the inverting input node of the op amp. Solve this equation for v_o.

4. Use the two power supply voltages as the limiting values for v_o and calculate the range of values for v_s that will keep v_o within its limiting values.

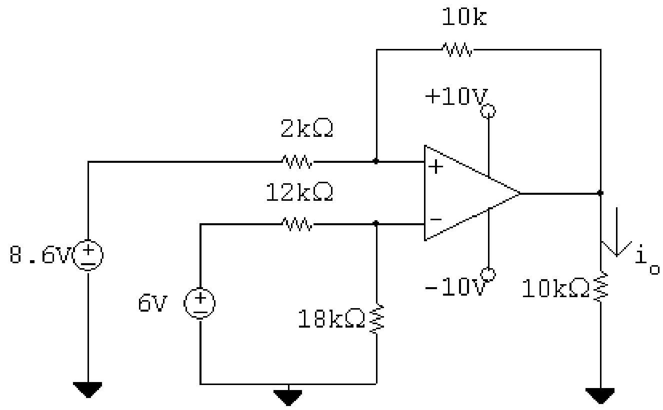

Figure 7: The circuit for Op Amp Practice Problem 3.

Op Amp Practice Problem 3

Calculate i_o for the circuit in Fig. 7.

1. Assume that the op amp is ideal and operating in its linear region. Label the three node voltages, two at the op amp's input and one at its output, in Fig. 7.

2. Calculate the value of the voltage at the non-inverting input node of the op amp, or write an equation for the voltage v_p in terms of the source voltage.

3. Write a node voltage equation at the inverting input node of the op amp. Solve this equation for v_o. Use this value of v_o to calculate i_o.

4. Is the value for v_o within the range of voltages defined by the power supplies? If so, your value of i_o is correct. If not, you must recalculate i_o based on the saturated value of v_o.

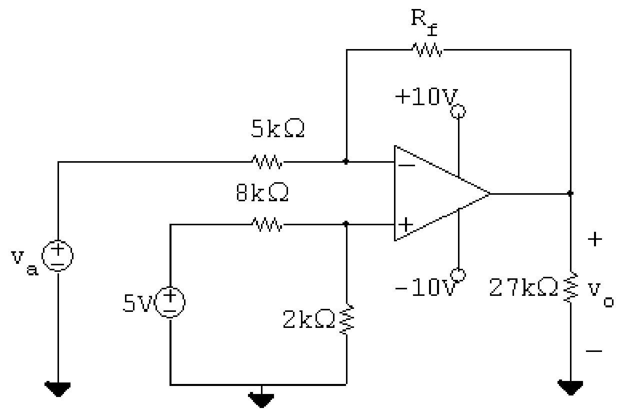

Figure 8: The circuit for Op Amp Practice Problem 4.

Op Amp Practice Problem 4

What value for R_f will yield the equation $v_o = 5 - 4v_a$ for the circuit in Fig. 8.

1. Assume that the op amp is ideal and operating in its linear region. Label the three node voltages, two at the op amp's input and one at its output, in Fig. 8.

2. Calculate the value of the voltage at the non-inverting input node of the op amp, or write an equation for the voltage v_p.

3. Write a node voltage equation at the inverting input node of the op amp. Simplify this equation for v_o. Use this equation for v_o to calculate R_f.

4. This problem does not concern a calculated value for v_o. We assume that the op amp is in its linear region of operation in order to obtain the equation specified in the problem.

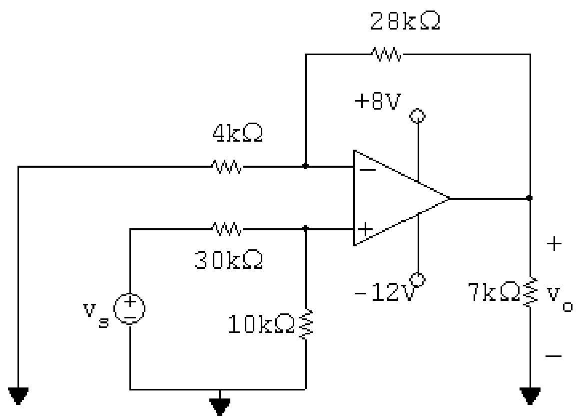

Figure 9: The circuit for Op Amp Practice Problem 5.

Op Amp Practice Problem 5

Find the range of values for the voltage v_s such that the output voltage v_o does not saturate and the op amp remains in its linear region of operation for the circuit in Fig. 9.

1. Assume that the op amp is ideal and operating in its linear region. Label the three node voltages, two at the op amp's input and one at its output, in Fig. 9.

2. Calculate the value of the voltage at the non-inverting input node of the op amp, or write an equation for the voltage v_p in terms of the source voltage.

3. Write a node voltage equation at the inverting input node of the op amp. Solve this equation for v_o.

4. Use the two power supply voltages as the limiting values for v_o and calculate the range of values for v_s that will keep v_o within its limiting values.

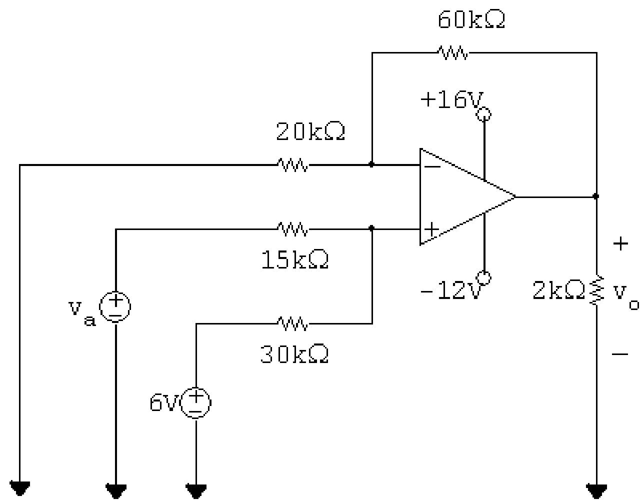

Figure 10: The circuit for Op Amp Practice Problem 6.

Op Amp Practice Problem 6

Find the range of values for the voltage v_a such that the output voltage v_o does not saturate and the op amp remains in its linear region of operation for the circuit in Fig. 10.

1. Assume that the op amp is ideal and operating in its linear region. Label the three node voltages, two at the op amp's input and one at its output, in Fig. 10.

2. Calculate the value of the voltage at the non-inverting input node of the op amp, or write an equation for the voltage v_p in terms of the source voltage.

3. Write a node voltage equation at the inverting input node of the op amp. Solve this equation for v_o.

4. Use the two power supply voltages as the limiting values for v_o and calculate the range of values for v_a that will keep v_o within its limiting values.

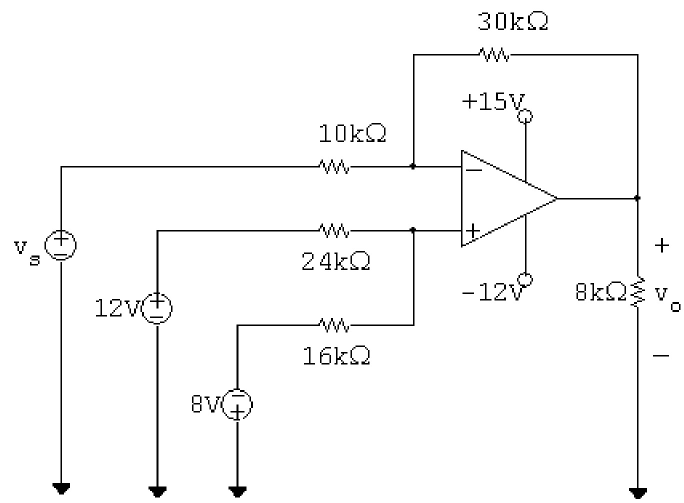

Figure 11: The circuit for Op Amp Practice Problem 7.

Op Amp Practice Problem 7

Find the range of values for the voltage v_s such that the output voltage v_o does not saturate and the op amp remains in its linear region of operation for the circuit in Fig. 11.

1. Assume that the op amp is ideal and operating in its linear region. Label the three node voltages, two at the op amp's input and one at its output, in Fig. 11.

2. Calculate the value of the voltage at the non-inverting input node of the op amp, or write an equation for the voltage v_p in terms of the source voltage.

3. Write a node voltage equation at the inverting input node of the op amp. Solve this equation for v_o.

4. Use the two power supply voltages as the limiting values for v_o and calculate the range of values for v_s that will keep v_o within its limiting values.

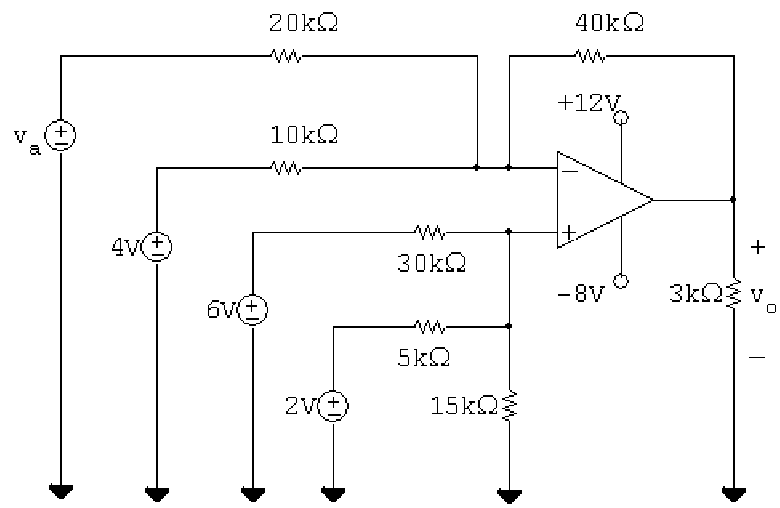

Figure 12: The circuit for Op Amp Practice Problem 8.

Op Amp Practice Problem 8

Find the range of values for the voltage v_a such that the output voltage v_o does not saturate and the op amp remains in its linear region of operation for the circuit in Fig. 12.

1. Assume that the op amp is ideal and operating in its linear region. Label the three node voltages, two at the op amp's input and one at its output, in Fig. 12.

2. Calculate the value of the voltage at the non-inverting input node of the op amp, or write an equation for the voltage v_p in terms of the source voltage.

3. Write a node voltage equation at the inverting input node of the op amp. Solve this equation for v_o.

4. Use the two power supply voltages as the limiting values for v_o and calculate the range of values for v_a that will keep v_o within its limiting values.

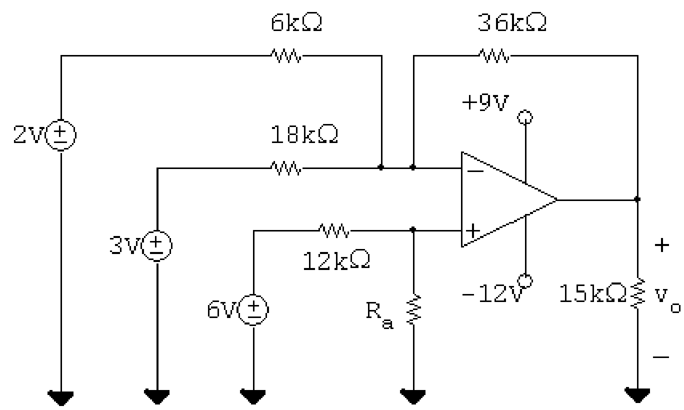

Figure 13: The circuit for Op Amp Practice Problem 9.

Op Amp Practice Problem 9

Find the range of values for the resistor R$_a$ such that the output voltage v_o does not saturate and the op amp remains in its linear region of operation for the circuit in Fig. 13.

1. Assume that the op amp is ideal and operating in its linear region. Label the three node voltages, two at the op amp's input and one at its output, in Fig. 13.

2. Calculate the value of the voltage at the non-inverting input node of the op amp, or write an equation for the voltage v_p.

3. Write a node voltage equation at the inverting input node of the op amp. Solve this equation for v_o.

4. Use the two power supply voltages as the limiting values for v_o and calculate the range of values for R_f that will keep v_o within its limiting values.

Reading

- in *Introductory Circuits for Electrical and Computer Engineering*:
  - Section 4.1 — terminology
  - Section 4.2 — op amp operating regions
  - Section 4.3 — inverting amplifier
  - Section 4.4 — summing amplifier
  - Section 4.5 — non-inverting amplifier
  - Section 4.6 — difference amplifier
- in *Electric Circuits*, sixth edition:
  - Section 5.1 — terminology
  - Section 5.2 — op amp operating regions
  - Section 5.3 — inverting amplifier
  - Section 5.4 — summing amplifier
  - Section 5.5 — non-inverting amplifier
  - Section 5.6 — difference amplifier
- Workbook section — Node Voltage Method

Additional Problems

- in *Introductory Circuits for Electrical and Computer Engineering*:
  - 4.2 — 4.3
  - 4.9 — 4.12
  - 4.21 — 4.24
  - 4.26
- in *Electric Circuits*, sixth edition:
  - 5.2 — 5.3
  - 5.10 — 5.14
  - 5.22 — 4.25
  - 5.27

Solutions

- Op amp Practice Problem 1: $4.5\text{ V} \leq v_s \leq 12\text{ V}$.
- Op amp Practice Problem 2: $-19.5\text{ V} \leq v_s \leq -12\text{ V}$.
- Op amp Practice Problem 3: $i_o = -1\text{ mA}$.
- Op amp Practice Problem 4: $\mathrm{R}_f = 20\text{ k}\Omega$.
- Op amp Practice Problem 5: $-6\text{ V} \leq v_s \leq 4\text{ V}$.
- Op amp Practice Problem 6: $-1.5\text{ V} \leq v_a \leq 3\text{ V}$.
- Op amp Practice Problem 7: $-5\text{ V} \leq v_s \leq 4\text{ V}$.
- Op amp Practice Problem 8: $-7\text{ V} \leq v_a \leq 3\text{ V}$.
- Op amp Practice Problem 9: $1.5\text{ k}\Omega \leq \mathrm{R}_f \leq 12\text{ k}\Omega$.